Lecture 8

Thursday, February 4, 2021 4:11 PM

* Prayer
* Spiritual thought
* Answering questions ...

About domains and level sets:

$$f(x,y) = \sqrt{x} + \sqrt{4 - 4x^2 - y^2}$$

$$g(x,y,z) = \sqrt{x} + \sqrt{4 - 4x^2 - y^2 - z^2}$$

Limit

(x_0, y_0)

$$\lim_{(x,y)\to(x_0,y_0)} f(x,y) = L$$

Ex

$$f(x,y) = \frac{xy}{x^2+y^2}$$

$\leadsto$ To show that the limit doesn't exist, we only need to show that the limits along two different paths are different.

$$f(x,y) = \frac{x^2 y}{x^2+y^2}$$

$\leadsto$ Compare the order of the numerator and the denominator.

[Use Squeeze theorem]

Mathematica:

Limit[xy/(x^2+y^2) , {x,y} → {0,0}]

Note:

$$\lim_{(x,y)\to(a,b)} f(x,y) \neq \lim_{x\to a} \lim_{y\to b} f(x,y)$$

Can you find an example?

$$f(x,y) = \frac{x}{y} \qquad \lim_{y\to 0} \lim_{x\to 0} f(x,y) = 0$$

$$\lim_{x\to 0} \lim_{y\to 0} f(x,y) \quad \text{DNE}$$

$$f(x,y) = \frac{x}{x+y} \qquad \lim_{y\to 0} \lim_{x\to 0} f(x,y) = 0$$

$$\lim_{x\to 0} \lim_{y\to 0} f(x,y) = 1.$$

Continuity:

A function $f: D \subset \mathbb{R}^n \to \mathbb{R}$ is said to be continuous at $A \in D$

if $\lim_{x\to A} f(x) = f(A)$.

Polynomials are continuous everywhere.

Rational functions are continuous where the denominators are nonzero.

Sum of two continuous functions is a continuous function.

Composition of two continuous function is a continuous function.

Ex:

$$\lim_{(x,y)\to(0,0)} \frac{x^2+2y^2}{3x^2+y^2} \qquad \lim_{(x,y)\to(1,2)} \frac{2x-y}{4x^2-y^2}$$

$$\lim_{(x,y)\to(1,2)} \frac{2x+y}{4x^2-y^2} \qquad \lim_{(x,y)\to(0,0)} xy\sin\left(\frac{1}{x^2+y^2}\right)$$

Partial derivatives

$f(x) \rightsquigarrow f'(x)$: rate of change of f with respect to x.

$f(x,y) \rightsquigarrow f_x, f_y$: rate of change of f with respect to x, y, respectively.

Ex: $f(x,y) = x^2 + xy + y^2$

Point $A(1,2)$.

$$f_x(1,2) = \left[f(x,2)\right]'\Big|_{x=1} = \left(x^2+2x+4\right)'\Big|_{x=1}$$

$$= (2x+2)\Big|_{x=1} = 4.$$

To visualize:

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ContourPlot3D[{z==x^2+xy+y^2, y==2}, {x,0,2}, {y,1,3},
    {z,1,20}]
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Ex (Clairaut's theorem)

Compute u_{xy} and u_{yx} of

$$u = x^3y^2 - y^3$$

$$u = \ln(x+2y)$$

$$u = x^y$$